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The Reference Ideal Method and the Pythagorean fuzzy numbers

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ABSTRACT

As it is well known, in spite of having small dimensions, there are daily many situations that require the solution of a decision-making problem: eating, streets crossing, assessments, shopping and so on. Generally, the way of working on these types of problems depends on how the information used to evaluate each alternative is provided and represented, as for instance is the case with: crisp values, fuzzy values, Pythagorean values, etc. In this way, different very well-known methods have been developed and modified to help to solve this kind of problems. Among them, the following may be remarked: AHP, PROMETHEE, ELECTRE, VIKOR, TOPSIS. But there are many other. This paper shows how to apply the so-called Reference Ideal Method (RIM), previously developed by the authors, when Pythagorean Fuzzy numbers are used to evaluate each alternative. The paper shows how to solve a decision-making problem through the proposed method using such kind of fuzzy numbers and, in order to show how to practically apply the RIM method, an illustrative example is provided.

1. Introduction

Multi-criteria decision-making (MCDM) methods are mathematical models that help the decision-maker to take decisions in scenarios where the possible alternatives are evaluated over multiple conflicting criteria.

A MCDM problem is characterized by a finite set of alternatives represented as $A=\{A_i \mid i=1,2,\ldots,m\}$, where m represents the number of m alternatives. The alternatives are evaluated according to certain criteria, denoted as $C=\{C_j \mid j=1,2,\ldots,n\}$, where n is the number of criteria. The criteria can have different domains, and may represent a cost (which is desirable to minimize) or a benefit (desirable to maximize).

In addition, each criterion is assigned an importance weight, represented as $W=\{w_j \mid j=1,2,\ldots,n\}$. These weights are normalized to add up to one, i.e., $\sum_{j=1}^{n} w_j = 1$ and this information is organized in a decision matrix $(M_{m\times n})$.

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$$M_{mxn} = \begin{array}{c} \\ \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} w_1 & w_2 & & w_n \\ C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1} & z_{m2} & \cdots & z_{mn} \end{pmatrix} \end{array}$$

where each element z_{ij} represents the value of the alternative A_i with respect to the criterion C_j.

The application areas of these methods are huge [10]. Examples can be found in different areas as: in supplier selection [18], technical evaluation of tenderers [11], evaluation of service quality [5, 20], in renewable energy [16], etc.

There are many MCDM methods in the literature, as The Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) [3,4], Analytic Hierarchy Process (AHP) [14], ELimination Et Choix Traduisant la REalité (ELECTRE) [13], The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [9] or the TOPSIS-ELECTRE III [15], VIšekriterijumsko KOmpromisno Rangiranje (VIKOR) [12] or the new Reference Ideal Method (RIM) [5,6]. TOPSIS and RIM operate calculating distances to "ideal" or "reference" points. We choose these methods for comparison because they have the same input and all of them rely on a normalization procedure. But, it is true that depending of the MCDM method applied, the solution could be different.

On the other hand, it is important in a decision-making problem to consider the type of information that is available. We are accustomed to valuations being crisp numbers, as Paul measures 175 cm, but we know that all the measuring devices have a certain error; for that reason, it would be more logical to say that Paul's height is in the average, or if we want to express the height with numbers that Paul's height is between 174 or 176 cm, etc (see Figure 1). Considering the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, Bellman and Zadeh [2] introduced the theory of fuzzy sets in the MCDM problems. They suggested that the decision maker could employ the membership function to express his or her preference about the membership degree of an alternative A_i with respect to a criterion C_j.

Atanassov [1] presented the concept of intuitionistic fuzzy set (IFS), which is characterized by a membership degree and a non-membership degree satisfying the condition that the sum of its membership degree and non-membership degree is equal to or less than 1.

Recently, Yager [17] introduced Pythagorean Fuzzy Set (PFS) characterized by a membership degree and a non-membership degree satisfying the condition that the square sum of its membership degree and non-membership degree is equal to or less than 1, which is a generalization of IFS or the extension of Garg [8].

From the Pythagoreans Numbers, Zhang and Xu [20] extend the TOPSIS method for this case of fuzzy numbers. In order to compare the TOPSIS and RIM methods for the case of the Pythagoreans Numbers, we will base ourselves on the problem proposed by these authors to contrast the results.

1.1. Overview of FS, IFS and PFS

In this subsection, we consider the fuzzy, intuitionistic and Pythagorean basic concepts.

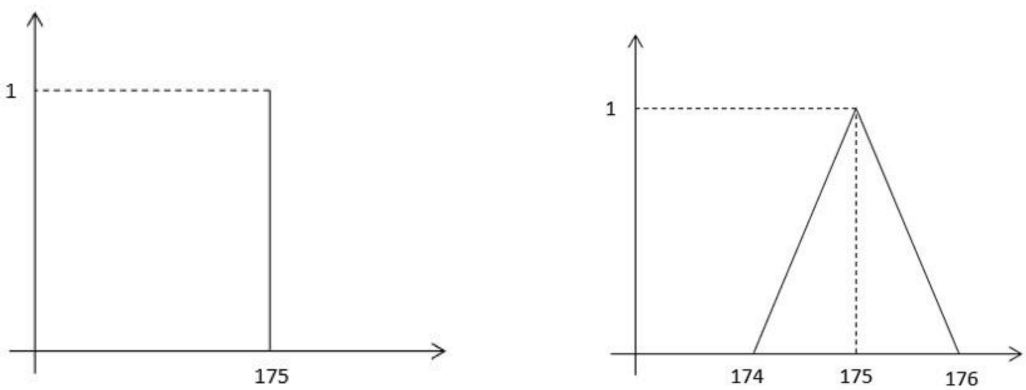
L.A. Zadeh in his seminal paper [19] write: "A fuzzy set (FS) is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one".

This theory was proposed in 1965 and it is based in the imprecision and subjectivity of the human reasoning. It is well known that human reasoning is not binary yes (true) or no (false) but imprecise, by them, an important way of modelling the imprecision is the introduction of the membership function.

Definition 1: If X is a collection of objects denoted generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs [18]:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) / x \in X \right\} \tag{1}$$

where $\mu_{\tilde{A}}(x)$ is called the membership function which maps X to the membership space M. Its range is the subset of nonnegative real numbers whose supremum is finite. In the case of $\sup \mu_{\tilde{A}}(x) = 1$ the fuzzy sets are normalized.



(a) (b)

Figure 1: Representation of the crisp number 175 cm (a) and the fuzzy number 175 cm (b).

Afterwards, Atanassov [1] in 1986 introduced the concept of intuitionistic fuzzy sets (IFS). The IFS was defined as an extension of the ordinary FS.

Definition 2: As opposed to a fuzzy set in X, given by $\tilde{A} = \left\{ \left(x, \mu_A(x) \right) / x \in X \right\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set $\tilde{A}$, an intuitionistic fuzzy set B is given by

$$B = \left\{ \left(x, \mu_B(x), \nu_B(x) \right) / x \in X \right\} \tag{2}$$

where $\mu_B : X \to [0,1]$ and $\nu_B : X \to [0,1]$ are such that:

$$0 \le \mu_B(x) + \nu_B(x) \le 1 \tag{3}$$

and $\mu_B, \nu_B \in [0,1]$ expresses the degrees of membership and non-membership of $x \in A$ and $x \in B$, respectively.

Definition 3: The definition is analogous to that of the IFS numbers but changes condition (3), which is modified by expression:

$$0 \le \left(\mu_B(x) \right)^2 + \left(\nu_B(x) \right)^2 \le 1 \tag{4}$$

The difference between IFS and PFS numbers is expressed in Figure 2.

As Yager has noted there are PFS that are not IFS. This is the case of the $\left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$ number. It is not an IFS because $\frac{\sqrt{3}}{2} + \frac{1}{2} > 1$, but it is PFS since $\left(\frac{\sqrt{3}}{2} \right) + \left(\frac{1}{2} \right) < 1$.

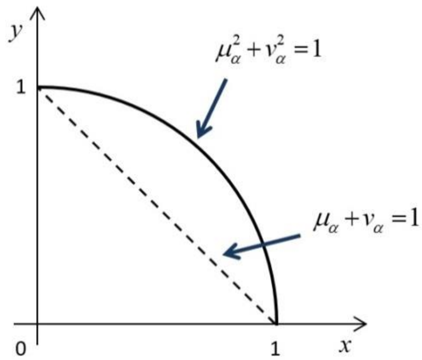


Figure 2: Representation of intuitionistic numbers $\mu_B(x)+v_B(x)\le 1$ and Pythagorean numbers $(\mu_B(x))^2+(v_B(x))^2\le 1$.

Considering the work of Yager, and the paper of Zhang and Xu [20], we will present a new solution for RIM [5] with Pythagorean fuzzy numbers. To do so, in Section 2, we develop a Pythagorean fuzzy RIM approach to solve the MCDM problems with PFSs. In Section 3, by means of a real problem we provide the practical decision-making, Section 4 presents our conclusions.

2. Pythagorean RIM

There are two important methods in MCDM as ViKOR and TOPSIS that incorporate the conception of the Positive Ideal solution (PIS) and the Negative Ideal Solution (NIS), based on the maximum value and/or the minimum value accordingly. But in practice, the ideal solution does not necessarily have to be one of the extreme values, but a value in between. For example, consider the case for selecting a researcher for an entity. If the age is one of the criteria being assessed, the person wanted should be between 30 and 40 years old in the ideal case. Assume the age range of our candidates is between 23 to 70 years old; it is then evident that the PIS and the NIS do not have to be 23 nor 70 years old. This problem is solved with RIM [5].

In order to adapt the RIM algorithm when the information is given with Pythagorean numbers, it is necessary to define a series of operations such as the following:

Definition 4: Let $\beta_1=P\left(u_{\beta_1},v_{\beta_1}\right)$ and $\beta_2=P\left(u_{\beta_2},v_{\beta_2}\right)$ be two Pythagorean fuzzy numbers then, the following relationship can be established [17, 20]:

$$\beta_1\le\beta_2 \Leftrightarrow u_{\beta_1}\le u_{\beta_2}, v_{\beta_1}\ge v_{\beta_2}$$

And we will say that, β_1 is better than β_2 .

Definition 5: Let $\beta_1=P\left(u_{\beta_1},v_{\beta_1}\right)$ and $\beta_2=P\left(u_{\beta_2},v_{\beta_2}\right)$ be two Pythagorean fuzzy numbers then, the distance between two PFSs is defined by [18, 5]:

$$dist(\beta_1,\beta_2)=\frac{1}{2}\left(\left|(u_{\beta_1})^2-(u_{\beta_2})^2\right|+\left|(v_{\beta_1})^2-(v_{\beta_2})^2\right|+\left|(\pi_{\beta_1})^2-(\pi_{\beta_2})^2\right|\right)$$

where $\pi_{\beta_1}=\sqrt{1-\mu_{\beta_1}^2-v_{\beta_1}^2}$ and $\pi_{\beta_2}=\sqrt{1-\mu_{\beta_2}^2-v_{\beta_2}^2}$, represent the degrees of indeterminacy.

Definition 6: Let $\beta_1=P\left(u_{\beta_1},v_{\beta_1}\right)$ and $\beta_2=P\left(u_{\beta_2},v_{\beta_2}\right)$ be two Pythagorean fuzzy numbers then, the Pythagorean fuzzy reference interval can be formed by $IRP=[P_1,P_2]$, where:

$$P_1 = P\left(\min\left\{\mu_{\beta_1},\mu_{\beta_2}\right\},\max\left\{v_{\beta_1},v_{\beta_2}\right\}\right)$$
$$P_2 = P\left(\max\left\{\mu_{\beta_1},\mu_{\beta_2}\right\},\min\left\{v_{\beta_1},v_{\beta_2}\right\}\right)$$

Another of the necessary operations is to determine the minimum distance to a Pythagorean fuzzy reference interval, in which case the following definition is required.

Definition 7: Let $\beta_x = P\left(u_{\beta_x}, v_{\beta_x}\right)$ be the Pythagorean fuzzy number and the Pythagorean fuzzy reference interval $IRP = \left[P_1, P_2\right]$, then the minimum distance from β_x to IRP is defined by:

$$\begin{aligned} &d_{\min} : \beta_x \otimes \left[P_1, P_2\right] \rightarrow [0,1] \\ &d_{\min}\left(\beta_x, \left[P_1, P_2\right]\right) = \min\left(dist\left(\beta_x, P_1\right), dist\left(\beta_x, P_2\right)\right) \end{aligned} \tag{5}$$

With the previous definitions, the conditions have been created to define the normalization function for working with Pythagorean fuzzy numbers.

Definition 8: Let $\beta_x = P\left(u_{\beta_x}, v_{\beta_x}\right)$ be the Pythagorean fuzzy number and $IRP = \left[P_1, P_2\right]$ the Pythagorean fuzzy reference interval, then the normalization function is defined by:

$$f : \beta_x \oplus IRP \rightarrow [0,1]$$

$$f\left(\beta_x, \left[P_1, P_2\right]\right) = \begin{cases} 1 & if \quad \beta_x \in \left[P_1, P_2\right] \\ 1 - \dfrac{d_{\min}\left(\beta_x, \left[P_1, P_2\right]\right)}{\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} \beta_x \in \left[(0,1), P_1\right] \vee \\ \beta_x \in \left[P_2, (1,0)\right] \end{pmatrix} \wedge D \\ 1 - \dfrac{dist\left(\beta_x, \left(\min\left\{\mu_{P_1}, \mu_{P_2}\right\}, \min\left\{v_{P_1}, v_{P_2}\right\}\right)\right)}{2\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} \mu_{\beta_x} \le \min\left\{\mu_{P_1}, \mu_{P_2}\right\} \wedge \\ v_{\beta_x} \le \min\left\{v_{P_1}, v_{P_2}\right\} \end{pmatrix} \wedge D \\ 1 - \dfrac{dist\left(\beta_x, \left(\max\left\{\mu_{P_1}, \mu_{P_2}\right\}, \max\left\{v_{P_1}, v_{P_2}\right\}\right)\right)}{2\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} \mu_{\beta_x} \ge \max\left\{\mu_{P_1}, \mu_{P_2}\right\} \wedge \\ v_{\beta_x} \ge \max\left\{v_{P_1}, v_{P_2}\right\} \end{pmatrix} \wedge D \\ 1 - \dfrac{d_{\min}\left(\left(\max\left\{\mu_{\beta_x}, \mu_{P_2}\right\}, v_{\beta_x}\right), \left[P_1, P_2\right]\right)}{\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} (P_1 < \beta_x) \wedge \\ (v_{\beta_x} < v_{P_2}) \end{pmatrix} \wedge D \\ 1 - \dfrac{d_{\min}\left(\left(\min\left\{\mu_{\beta_x}, \mu_{P_1}\right\}, v_{\beta_x}\right), \left[P_1, P_2\right]\right)}{\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} (\beta_x < P_2) \wedge \\ (v_{P_1} < v_{\beta_x}) \end{pmatrix} \wedge D \\ 1 - \dfrac{d_{\min}\left(\left(\mu_{\beta_x}, \max\left\{v_{\beta_x}, v_{P_1}\right\}\right), \left[P_1, P_2\right]\right)}{\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} (\beta_x < P_2) \wedge \\ (v_{\beta_x} < v_{P_1}) \end{pmatrix} \wedge D \\ 1 - \dfrac{d_{\min}\left(\left(\mu_{\beta_x}, \min\left\{v_{\beta_x}, v_{P_2}\right\}\right), \left[P_1, P_2\right]\right)}{\max\left(dist\left((0,1), P_1\right), dist\left(P_2, (1,0)\right)\right)} & if \quad \begin{pmatrix} (P_1 < \beta_x) \wedge \\ (v_{P_2} < v_{\beta_x}) \end{pmatrix} \wedge D \end{cases} \tag{6}$$

where $\beta_x \in \left[P_1, P_2\right] \Leftrightarrow P_1 \le \beta_x, \beta_x \le P_2$ and $D = \left(dist\left((0,1), P_1\right) \ne 0 \vee dist\left(P_2, (1,0)\right) \ne 0\right)$.

2.1 The RIM algorithm with Pythagorean numbers

On the basis of the concepts referred to above, the steps of the RIM method for working with Pythagorean fuzzy numbers shown below.

Step 1**:** Define the work context.

In this step the conditions in the work context are established, and for each criterion C_j the following aspects are defined:

a) The Reference Ideal $IRP = [P_1, P_2]$.
b) The weight w_j associated to each criterion.

Step 2**:** Obtain the valuation matrix V, in correspondence with the defined criteria. In this case, β_{ij} represent a Pythagorean fuzzy number.

$$V = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1} & \beta_{m2} & \cdots & \beta_{mn} \end{pmatrix}$$

Step 3**:** Normalize the valuation matrix V with the reference ideal.

$$N = \begin{pmatrix} f(\beta_{11}, IRP_1) & f(\beta_{12}, IRP_2) & \cdots & f(\beta_{1n}, IRP_n) \\ f(\beta_{21}, IRP_1) & f(\beta_{22}, IRP_2) & \cdots & f(\beta_{2n}, IRP_n) \\ \vdots & \vdots & \ddots & \vdots \\ f(\beta_{m1}, IRP_1) & f(\beta_{m2}, IRP_2) & \cdots & f(\beta_{mn}, IRP_n) \end{pmatrix}$$

where, f is the function considered in (6).

Step 4**:** Calculate the weighted normalized matrix P, though.

$$P = N \otimes W = \begin{pmatrix} n_{11} \cdot w_1 & n_{12} \cdot w_2 & \cdots & n_{1n} \cdot w_n \\ n_{21} \cdot w_1 & n_{22} \cdot w_2 & \cdots & n_{2n} \cdot w_n \\ \vdots & \vdots & \ddots & \vdots \\ n_{m1} \cdot w_1 & n_{m2} \cdot w_2 & \cdots & n_{mn} \cdot w_n \end{pmatrix}$$

Step 5**:** Calculate the variation to the normalized reference ideal for each alternative A_i.

$$A_i^+ = \sqrt{\sum_{j=1}^{n} \left(p_{ij} - w_j \right)^2} \text{ and } A_i^- = \sqrt{\sum_{j=1}^{n} \left(p_{ij} \right)^2}$$

where $i = 1, 2, ..., m$, $j = 1, 2, ..., n$ and p_{ij} are the values in matrix P.

Step 6**:** Calculate the relative index of each alternative A_i, through the expression.

$$R_i = \frac{A_i^-}{A_i^+ + A_i^-}, \quad 0 \le R_i \le 1, \qquad i = 1, 2, ..., m$$

***Step 7*:** Rank the alternatives A_i descending order. The alternatives that are in the top constitute the best solutions.

3. Illustrative Example

In order to show how to apply the RIM method with Pythagorean numbers, we will use the decision problem presented by Zhang and Xu [20] in the paper "Extension of TOPSIS to Multiple Criteria Decision Making with Pythagorean Fuzzy Sets". This decision problem consists of a study carried out by the Civil Aviation Administration of Taiwan (CAAT) to determine the best domestic airline of the four major Taiwan Airlines.

In this multicriteria decision problem, we have four alternatives referring to Taiwanese domestic airlines, which are shown below [19]:

- UNIAir (X_1)
- Transasia (X_2)
- Mandarin (X_3)
- Daily Air (X_4)

The alternatives (Taiwanese domestic airlines) referred to above, are evaluated for the following criteria:

- Booking and ticketing service (C_1)
- Check-in and boarding process (C_2)
- Cabin service (C_3)
- Responsiveness (C_4)

***Step* 1:** To apply the RIM-P method, it is first necessary to define the work context. In this case, Table 1 shows the different criteria and their respective reference ideal and weights. The working range is between $P_1(0,1)$ and $P_2(1,0)$.

Table 1: Definition of the working context

Criteria	**Weights**	**Reference Ideal** *IRP*
C_1	0.15	$[(0.9,0),(0.9,0)]$
C_2	0.25	$[(0.9,0),(0.9,0)]$
C_3	0.35	$[(0.8,0),(0.8,0)]$
C_4	0.25	$[(0.7,0),(0.7,0)]$

***Step* 2:** In this case, the Pythagorean Fuzzy Decision Matrix can be obtained (see Table 2).

Table 2: Pythagorean fuzzy decision matrix

Alternatives	C_1	C_2	C_3	C_4
X_1	P(0.9,0.3)	P(0.7,0.6)	P(0.5,0.8)	P(0.6,0.3)
X_2	P(0.4,0.7)	P(0.9,0.2)	P(0.8,0.1)	P(0.5,0.3)
X_3	P(0.8,0.4)	P(0.7,0.5)	P(0.6,0.2)	P(0.7,0.4)
X_4	P(0.7,0.2)	P(0.8,0.2)	P(0.8,0.4)	P(0.6,0.6)

Step 3: Obtaining of the normalized valuation matrix (see Table 3).

Table 3: Normalized valuation matrix

Alternatives	C_1	C_2	C_3	C_4
X_1	0.91	0.64	0.36	0.87
X_2	0.35	0.96	0.99	0.76
X_3	0.83	0.68	0.72	0.84
X_4	0.68	0.83	0.84	0.64

Step 4**:** Obtaining of the weighted normalized valuation matrix (see Table 4).

Table 4: Weighted normalized matrix

Alternatives	C_1	C_2	C_3	C_4
X_1	0.1365	0.16	0.126	0.2175
X_2	0.0525	0.24	0.3465	0.19
X_3	0.1245	0.17	0.252	0.21
X_4	0.102	0.2075	0.294	0.16

Step 5**:** Calculation of the R_i index (see Table 5).

Table 5: Indexes calculation

Alternatives	A_i^+	A_i^-	R_i
X_1	0.24396	0.32774	0.57328
X_2	0.11497	0.46532	0.80187
X_3	0.13511	0.38988	0.74264
X_4	0.12388	0.40681	0.76657

Step 6**:** Obtaining of the final ranking (see Table 6).

Table 6: Indexes calculation

Alternatives	A_i^+	A_i^-	R_i
X_1	0.24396	0.32774	0.57328
X_2	0.11497	0.46532	0.80187
X_3	0.13511	0.38988	0.74264
X_4	0.12388	0.40681	0.76657

From the relative index, the alternatives are ordered as follows $X_2 > X_4 > X_3 > X_1$.

When comparing with the results obtained in [5], it is observed that the results obtained by the three methods coincide both in the best alternative (Aerolinea Transasia) and in the worst alternative (Daily Air). The change occurs in the second and third positions among the method of Zhang and Xu [20] and the other two methods Yager [17] and RIM [5] that do coincide (see Table 7).

Table 7: Indexes calculation

Method	The alternative Order
Yager's method [17]	$X_2 > X_4 > X_3 > X_1$
Zhang and Xu [20]	$X_2 > X_3 > X_4 > X_1$
RIM with Pythagorean numbers [5]	$X_2 > X_4 > X_3 > X_1$

4. Conclusions

The modification of the different methods used to solve decision-making problems using Pythagorean numbers is very useful, since it allows us to expand this group of tools and thus face particular solutions with greater objectivity. In this case, it was possible to modify the RIM method, so that it could operate with the Pythagorean Fuzzy Numbers, for it was only necessary to modify the first step of the algorithm, as well as the normalization function.

Through the illustrative example, it can be seen that the RIM solution is identified with the Yager solution and varies with that of Zhang and Xu [20], in which the alternatives of the second and third positions are exchanged.

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References

1. Atanassov, K.T. (1986), Intuitionistic fuzzy sets, *Fuzzy sets and Systems*, 20(1), 87-96.
2. Bellman, R. E., Zadeh, L. A. (1970), Decision making in a fuzzy environment, *Management Science* 17, 141-164.
3. Brans, J.P., Mareschal, B., Vincke, P. (1984), Promethee: a new family of outranking methods in multicriteria analysis, In: *Operational Research* '84, Proceedings of the Tenth International Conference, pp. 477–490. Washington, DC.
4. Brans, J.P., Vincke, Ph, Mareschal, B. (1986), How to select and how to rank projects: *the Promethee method*, *European Journal of Operations Research* 24(2), 228–238.
5. Cables, E., Lamata, M.T., Verdegay, J.L (2016), RIM-reference ideal method in multicriteria decision making, *Information Sciences* 1-10.
6. Ceballos, B., Lamata, M.T., Pelta, D, (2017). Fuzzy Multicriteria Decision-Making Methods: A Comparative Analysis. *International Journal of Intelligent Systems*, 32, 722–738.
7. Chen, S.J., Hwang, C.L., Hwang, F.P. (1992), Fuzzy multiple attribute decision making (methods and applications), *Lecture Notes in Economics and Mathematical System* (pp. 1-536). Springer-Verlag Berlin Heidelberg.
8. Garg, H (2018), Some methods for strategic decision-making problems with immediate probabilities in Pythagorean fuzzy environment, *International Journal of Intelligent Systems* 33(4), 687 – 712.
9. Hwang, C.L. and Yoon, K. (1995), *Multiple Attribute Decision Making: An Introduction*, SAGE Publications, California.
10. Mardani, A., Jusoh, A., Zavadskas, E. K. (2015). Fuzzy multiple criteria decision-making techniques and applications: two decades review from 1994 to 2014, *Expert Systems Applications* 42(8), 4126–4148.
11. Mohamed, N.H., Lamsali, H., Sathyamoorthy, D. (2015), Multi-criteria decision making (mcdm) for technical evaluation of tenderers: a review of methods employed, *Defence S&T Technical Bulletin* 8(1), 90–102.

12. Opricovic, S., Tzeng, G.H. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS, *European Journal of Operational Research* 156(2), 445–455.
13. Roy, B., Hugonnard, J.C, (1982), Ranking of suburban line extension projects on the Paris metro system by a multicriteria method, *Transportation Research Part A: General* 16(4), 301–312.
14. Saaty, T.L. (1980), *The Analytic Hierarchy Process*, McGraw-Hill, New York.
15. Sánchez-Lozano, J.M., García-Cascales, M.S., Lamata,M.T. (2016), Comparative TOPSIS-ELECTRE TRI methods for optimal sites for photovoltaic solar farms. Case study in Spain, *Journal of Cleaner Production* 127, 387-398.
16. Taha, A., R., Daim, T, (2013). Multi-criteria applications in renewable energy analysis, a literature review. In: Daim, T., Oliver, T., Kim, J. (eds.) *Research and Technology Management in the Electricity Industry: Methods, Tools and Case Studies* (pp 17–30). Springer, London.
17. Yager, R., Ali M. Abbasov, A. M. (2013), Pythagorean Membership Grades, Complex Numbers, and Decision Making, *International Journal of Intelligent Systems* 28(5), 436–452.
18. Yildiz, A., Yayla, A.Y. (2015), Multi-criteria decision-making methods for supplier selection: a literature review, *The South African Journal of Industrial Engineering* 26(2), 158–177.
19. Zadeh, L.A. (1965), Fuzzy sets, *Information and Control* 8, 338-353.
20. Zhang, X., Xu, Z. (2014), Extension of TOPSIS to Multiple Criteria Decision Making with Pythagorean Fuzzy Sets, *International Journal of Intelligent Systems* 29, 1061–1078.