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The Application of Data Envelopment Analysis in Fuzzy Queuing Models

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ABSTRACT

In this paper, an approach is presented for the evaluation of efficiency in fuzzy queuing models with publicity and renouncement. In the existing method proposed for the functions of fuzzy profit of queuing models, in the last stage the function of standardized profit and the level of expense can be evaluated among different α-level set. According to the new approach we determine which α-level set can be chosen for the system as efficient and ideal. In this step an interval data envelopment analysis model is suggested to get the overall efficiency of the proposed method for the functions of fuzzy profit of queuing models. Numerical illustration is provided to show the application of interval DEA models to the fuzzy queuing systems.

1. Introduction

There exist two large kinds of queuing models, one of them is the descriptive model which indicates the mean values and the probabilities of the performance measures in the system, that the patterns of arrivals and services, the number of servers, the capacity of the system and the queue discipline are fixed. The other one is normative model which considers optimum setting target for the system namely model of design and queues decision model. They usually control the measures that refer to services, such as service rate, number of servers, queue discipline or some of their combinations. Therefore, the arrival of customers to the system can be under control, whether they are increased or decreased, they can be assigned to different types of servers or they can even be regulated with some kind of toll [24]. In the literature many researches have been considered that analyses design and control problems associated to the classical queuing models. Armstrong [3] and Jain [20] applied queuing models to the machine repair system with policies. Palunčić et al. [31] represented a comprehensive survey about the relation between queuing models and cognitive radio networks. Besides, Wang and Ke [33] suggested queuing models for repairable system with warm standbys. Al-Seedy [1, 2] proposed queuing models with fixed channels and the non-truncated queue m/m/2 with balking variable channels. Drekic and Wooldford [16], Goto et al. [19] and Sanga and Jian [29] utilized various queuing models in different fields.

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Pardo and Fuente [24] demonstrated that in classic queuing theory, the time between the arrivals and the service duration can be determined by the probability distributions. However, in the system the arrival pattern and the service pattern can be determined in a qualitative manner, such as fast, slow or moderate, rather than by probability distributions. Hence, fuzzy queuing models can be used in real problems than the traditional models. Previous researches, based on possibility and fuzzy queueing theory, are Buckley [5], Buckley et al. [7], Chen [9], Zhang and Phillips [37], Pardo and Fuente [24], Fuente and Pardo [25].

Pardo and Fuente [24], proposed a model of design and control of queues in the framework of fuzzy queuing theory in which the parameters exerted in the distributions of the service time are presented in the form of fuzzy numbers. They considered a situation in which the manager (or the decision maker) of some facilities wants to maximize the profit with the determination of the rate to be paid by every customer for his service and the level of publicity. They also proposed a new technique to optimize the functions of fuzzy profit of queuing models. Besides, the consideration of fuzzy environment to the queuing decision models results to application of their approach.

Based on the explanations mentioned above, in this paper we proposed an approach for evaluating the overall efficiency of the Pardo and Fuente method. In their method, in the last stage, they evaluate the function of standardized profit, level of expense in publicity and average arrival rate to the system in word of mouth model among different α-level set. So in the last stage, the main question may need to be answered is that which α-level set is working more efficiently. In this case, the manager or the decision maker (DM) can choose which α-level set is optimum and efficient for the system. The application of data envelopment analysis starts in this stage as we consider each α-level as DMU to be evaluated. To deal with this, interval DEA models have been chosen for evaluating the efficiency and also setting targets for the managers to decide the most efficient way.

Previously, Giokas and Pentzaropoulos [18] established two stages method for evaluating the relative operational efficiency of large-scale computer networks, in the first stage analytical results for the main performance indicators were obtained by a queuing model (M/M/I/K) of a typical network and the results were used, in the second stage, as a starting point for the application of a data envelopment analysis (DEA) procedure for getting the relative efficiency and improving the efficiency level of inefficient nodes, via input-oriented CCR dual model [8]. Malekmohammadi and Zainuddin [21] improved their method by presenting a target model [22] instead of CCR dual model.

This paper proceeds as follows. In Section 2 fuzzy queuing model is introduced. In Section 3 the proposed model for evaluation of fuzzy queuing system is presented. Numerical example and conclusion are discussed in Sections 4 and 5, respectively.

2. Fuzzy queuing model with publicity and renouncement

Two kinds of fuzzy queuing model with publicity and renouncement are, basic model and word of mouth model. In this paper, just the second one is considered. More details about basic models are mentioned in [24]. Now, we explain the Fuente and Pardo [24] approach into the Word of mouth model.

2.1. Word of mouth model

Now, we investigate a case in which a manager resolves on the optimum use of a service. Suppose the case which the clients, by Poisson process with rate $\tilde{\lambda}$, register for only one service. Each user values for the service provided by the system in R monetary units and pays F monetary units for the cost of service (Atkinson [4]). It has to be mentioned average service time $\tilde{t}$ is only known in an ambiguous manner which is introduced by fuzzy number.

The fuzzy model pertains to the manner of the customers in the system in which the arrival of clientele to the facilities is accomplished due to a Poisson process with $\tilde{\lambda}$ rate. The entrance of a customer in the system depends on first, the number of customers in the system, second the price F of the service, third obtaining some profit for him from the service. The objective is to calculate the arrival rate$\tilde{\lambda}$ and the price F which must be considered by the manager to maximize his profit. For an established price of service F, high cost in publicity has a negative influence on the arrival rate, due to the longitude of the customers in queue. Therefore, if the probability of admittance into the system is $\tilde{P}_{ad}$ (that is, the probability that the client will not balk in a fuzzy manner), high levels of publicity cause to a big quantity of customers who avoid incorporating into the system. According to the above definitions, the level of expense in publicity in word of mouth model $\tilde{P}_{Mw}$introduced with fuzzy number of the type is as follows:

$$\tilde{P}_{Mw} = \frac{\tilde{\lambda}}{\tilde{P}_{ad}} \tag{1}$$

The queuing model can be adapted to a fuzzy model which can be represented in the same notation as Kendall $M/\tilde{M}/1/k$. We have to mention that the Fuzzy probability of entry into the system $\tilde{P}_{ad}$ in a model $M/\tilde{M}/1/k$ is explained completely in [24].

2.2. The problem of fuzzy optimization

Fuente and Pardo [24] proposed a new method along with the work of Rommel anger et al. [2] to find the optimum in functions with fuzzy parameters. This method can carry out a great number of operations and calculations and also demonstrate the efficiency as validity.

2.2.1. Pardo and Fuente method:

In the fuzzy word of mouth model the publicity is expressed with fuzzy number. The fuzzy standardized profit is:

$$\tilde{S}_{Mw}(\tilde{\rho}, k) = \tilde{\rho}(1 - \frac{k}{R})\frac{1-\tilde{\rho}^{k}}{1-\tilde{\rho}^{k+1}} - \frac{\tilde{\rho}(1-\tilde{\rho}^{k+1})}{R(1-\tilde{\rho}^{k})} \tag{2}$$

where $\tilde{\rho}$ is traffic intensity ($\tilde{\rho} = \tilde{\lambda}\tilde{t}$);$k$, the capacity of the system; $\tilde{S}_{Mw}$ is unimodal function for $\tilde{\rho} \geq 0$, and they optimized it regard with $\tilde{\rho}$ utilizing methods of nonlinear optimization of bisection interval; R, the assessment of each user for the service prepared by the system.

We have to mention that the discrete function of standardized profit depending on k, $S_{Mw}(\rho_k, k)$when $k = 1,2,\dots,[R]$, is also unimodal function on k and their optimum value is gained in a value k close to the unity. In this model:

$$\tilde{P}_{Mw} = \frac{\tilde{\lambda}}{\tilde{P}_{ad}} = \tilde{\lambda}\frac{1-(\tilde{\lambda}\tilde{t})^{k+1}}{1-(\tilde{\lambda}\tilde{t})^{k}} \tag{3}$$

So, with the membership functions $\mu_{\tilde{\lambda}}(\lambda)$ and $\mu_{\tilde{t}}(t)$ and using the Zadeh's extension principle [21] the membership function of the level of expense in publicity is:

$$\mu_{\tilde{P}_{Mw}}(P) = \text{supmin}\{\mu_{\tilde{\lambda}}(\lambda), \mu_{\tilde{t}}(t) / P_{Mw} = \lambda\frac{1-(\lambda t)^{i+1}}{1-(\lambda t)^{i}}\} \tag{4}$$

and the membership function of the fuzzy standardized profit function is:

$$\mu_{\tilde{S}_{Mw}}(S_{Mw}) = \text{supmin}\{\mu_{\tilde{\lambda}}(\lambda), \mu_{\tilde{t}}(t) / S_{Mw} \tag{5}$$

$$= (\lambda, t, i) = \lambda t(1 - \frac{i}{R})\frac{1-(\lambda t)^{i}}{1-(\lambda t)^{i+1}} - \frac{\lambda t(1-(\lambda t)^{i+1})}{R(1-(\lambda t)^{i})}\}.$$

3. Data envelopment analysis and fuzzy queuing models

Data envelopment analysis (DEA) regarded as a technique for evaluating the relative efficiency of decision making units (DMUs). The first DEA model (CCR model), introduced by Charnes et al. [8] assumed for exact data, after that Cooper et al. [10] were pioneer researchers in presenting the DEA models in which the data were imprecise. In imprecise data envelopment analysis (IDEA) models the data are presented in the form of ordinal, interval and fuzzy which are usually presented in non-linear models. Cooper et al. [11] proposed some methods to convert the nonlinear model to a linear one. Zhu [38, 39] on the other hand shows that the non-linear IDEA can be solved in the standard linear CCR model via identifying a set of exact data from the imprecise input and output data. Despotis and Smirlis [14] discussed the problem of IDEA by converting a nonlinear DEA model to an equivalent linear problem by transforming only on the variables which results in interval efficiency scores. Wang et al. [34]

presented a new pair of interval DEA models which rectified the shortage of the previous interval efficiency. They presented the efficiency score by an interval bounded (best lower and upper bound of efficiency) for each DMU, as follows:

Let $j = 1,2,\dots,n,$ be indexes for DMUs; $i = 1,2;\dots,m,$ be index for inputs; $k = 1,2,\dots,p,$ be index for outputs; x_{ij}, amount of input i consumed by DMU $_j$; y_{kj}, quantity of output k produced by DMU $_j$. It is clear that θ should also be an interval number, which showed by $[\theta^L,\theta^U]$. Also, suppose that all the input and output data x_{ij} and y_{rj} $(i = 1,\dots,m; k = 1,\dots,p; j = 1,\dots,n)$ are located within the upper and lower bounds represented by the intervals $[x_{ij}^L, x_{ij}^U]$ and $\left[y_{kj}^L, y_{kj}^U\right]$ where $x_{ij}^L \geq 0$ and $y_{kj}^L \geq 0$.Also jo is the DMU under decision (usually denoted by DMU_{jo}); u_k and $v_i$are the weights assigned to the outputs and inputs; θ_{jo}^U stands for the best possible relative efficiency achieved by $DMU_{jo}$when all the DMUs are in the state of best production activity, while θ_{jo}^Lstands for the lower bound of the best possible relative efficiency of DMU_{jo}. They constituted a possible best relative efficiency interval $[\theta_{jo}^L, \theta_{jo}^U]$. besides, they presented the pair of interval DEA models in order to refrain from using the different production frontiers for measuring the efficiencies of different DMUs, as follows:

$$
\begin{aligned}
\max \quad & \theta_{jo}^U = \sum_{k=1}^{p} u_k y_{kjo}^U \\
s.t \quad & \sum_{i=1}^{m} v_i x_{ijo}^L = 1, \\
& \sum_{k=1}^{p} u_k y_{kj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0, \; j = 1,..,n \\
& u_k, v_i \geq 0 \qquad \forall i,k.
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\max \quad & \theta_{jo}^L = \sum_{k=1}^{p} u_k y_{kjo}^L \\
s.t \quad & \sum_{i=1}^{m} v_i x_{ijo}^U = 1, \\
& \sum_{k=1}^{p} u_k y_{kj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0, \; j = 1,..,n \\
& u_k, v_i \geq 0 \qquad \forall i,k.
\end{aligned} \tag{7}
$$

By Model (6) the production frontier for all the DMUs is clarified and by model (7) the production frontier can be utilized as a benchmark in order to measure the lower bound efficiency of each DMU. More information in [34].

A DMU, DMU $_o$, is said to be DEA efficient if its best possible upper bound efficiency $\theta_{jo}^{*U} = 1$; otherwise, it is said to be DEA inefficient if $\theta_{jo}^{*U} < 1$. (see Wang et al., [34]). Now, we consider the dual form of Wang models. Model (8) and (9) show the dual form of Model (6) and (7) respectively with an extra convexity constraint of $\sum_{j=1}^{n} \lambda_j = 1$(this is related to Variable Returns to Scale (Cooper et al. [12]).

$$
\begin{aligned}
\min \quad & \theta_{jo}^L \\
s.t. \quad & \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^L \leq \theta^L x_{ijo}^U, \qquad \forall i \\
& \sum_{j=1}^{n} \lambda_j y_{kj}^U \geq y_{kjo}^L, \qquad \forall k \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \theta_{jo}^L \quad free.
\end{aligned} \tag{8}
$$

$$\begin{aligned}
&\min \quad \theta_{jo}^{U} \\
&s.t. \\
&\sum_{j=1}^{n} \gamma_j x_{ij}^{L} \le \theta_{jo}^{U} x_{ijo}^{L}, \qquad \forall i \\
&\sum_{j=1}^{n} \gamma_j y_{kj}^{U} \ge y_{kjo}^{U}, \qquad \forall k \\
&\sum_{j=1}^{n} \gamma_j = 1, \\
&\gamma_j \ge 0, \theta_{jo}^{U} \quad free
\end{aligned} \tag{9}$$

We call Model (9), input -oriented Wang envelopment model. After solving Model (9) the corresponding vector $(\gamma_1^*, \gamma_2^*, \ldots, \gamma_n^*)$ defines for each DMU_{jo} as the operating point which it should aim. $\theta_{\mathrm{jo}}^{\mathrm{U}}$ can be used as a criterion for evaluating the relative efficiency for DMUs. Therefore, we can compute the target inputs and outputs of such a point from the formulation below:

$$\hat{x}_{ij} = \sum_{j=1}^{n} \gamma_j^* x_{ij}^{L},\ \forall i, \quad \hat{y}_{kj} = \sum_{j=1}^{n} \gamma_j^* y_{kj}^{U},\ \forall k, \tag{10}$$

By the above formulation, the DM or manager can set targets for the system as the models (8) and (9) can definitely compute the efficient DMU. The manager can also find which DMU is more efficient and ideal for the system and how the inefficient DMUs will improve themselves to be efficient as well. Many comprehensive researches with applications can be found in [30], [35] and [32].

3.1. Our approach

Now we will explain the application of Models (8) and (9) for fuzzy queueing model especially for Fuente and Pardo method. As it is clear in the last stage they computed the function of standardized profit, level of expense in publicity and average arrival rate to the system in word of mouth model among different $\alpha -$level set. The decision maker (DM) or manager may desire to know which $\alpha -$level set are better and suitable to be chosen for the system. If we suppose each $\alpha -$level set as DMU which has its own inputs or outputs we can have the following criteria:

$I_1 = \tilde{t}$: average service time

$I_2 = \tilde{\lambda}$: average arrival rate to the system

$O_1 = P_{Mw_\alpha}$:level of expense in publicity in word of mouth model or optimum value of the arrival rate of the customers into the system.

$O_2 = S_{Mw_\alpha}$:function of standardized profit in word of mouth model.

We can deal these inputs and outputs to Models (8) and (9) for evaluating the relative efficiency and setting target for each $\alpha -$level set as a DMU. By applying these two models, in the next section, we will show which $\alpha -$level set works effciently.

4. Numerical Results

In this section, we will illustrate our approach toward the application of interval DEA models to fuzzy queueing models. Table 1 shows the data collected from [24]. The 5 α-cuts ($\alpha = 0, \alpha = 0.25, \alpha = 0.5, \alpha = 0.75, \alpha = 1$) have been considered as a DMU to be evaluated. The average service time is described in the form of the triangular fuzzy number $\tilde{t} = [3,4,5]$ per unit of time. The $\tilde{t}$ can be converted into the interval data via the formulation:

$$t_{\alpha_j} = [\underline{t}_{\alpha_j}, \bar{t}_{\alpha_j}] = [\alpha_j + 3, 5 - \alpha_j], j = 1,2,3,4,5. \tag{11}$$

Each user assesses for the service prepared by the system in $R = 15$ monetary units. In this condition, the price F of the service can be between 1 and 15. In the word of mouth model, the non-fuzzy standardized profit function will be optimized as follows:

$$S_{Mw}(\rho, k) = \rho(1 - \frac{k}{15})\frac{1-\rho^k}{1-\rho^{k+1}} - \frac{\rho(1-\rho^{k+1})}{15(1-\rho^k)}$$

where $k = 1,2,\dots,15.$ By using methods of nonlinear optimization of bisection interval, the maximum of the function S_{Mw} and the corresponding value for the different values of k can be calculated. Fuente and Pardo (by comparing with the results) find that the maximum of the S_{Mw} and the corresponding value ρ are $S_{Mw} = S_{Mw}(\rho_3, 3) = 0.5364$ with $\rho_3 = 1.3254$ and the optimum solution can be obtained by $F^* = 12$ monetary units. By computing the values $k = 3$ and ρ_3, they calculated by α-cuts the membership functions $\tilde{\lambda}$and $\tilde{S}_{Mw}$. The α-cuts $\lambda_\alpha = \frac{\rho_3}{t_\alpha}$ are obtained by the Bucley and Qu's method [23] and the α-cuts S_{Mw} are calculated by applying the Dong and Shah's vertex method [15] and Dubois et al. [17] in the function:

$$S_{Mw_\alpha} = (\lambda, t) = \lambda_\alpha t_\alpha (1 - \frac{3}{15})\frac{1-(\lambda_\alpha t_\alpha)^3}{1-(\lambda_\alpha t_\alpha)^4} - \frac{\lambda_\alpha t_\alpha (1-(\lambda_\alpha t_\alpha)^4)}{15(1-(\lambda_\alpha t_\alpha)^3)}$$

The fuzzy level of publicity obtained as follows:

$$\widetilde{P}_{Mw} = \tilde{\lambda}\frac{1-(\tilde{\lambda}\tilde{t})^4}{1-(\tilde{\lambda}\tilde{t})^3}$$

The membership function of the fuzzy variable $\widetilde{P}_{Mw}$ through the use of α-cuts, for the 5 α-cuts can be obtained by using the Bucley and Qu's method [23]. Every α-cut has the expression:

$$P_{Mw\alpha} = [\underline{\lambda}_\alpha \frac{1-(\underline{\lambda}_\alpha \underline{t}_\alpha)^4}{1-(\underline{\lambda}_\alpha \underline{t}_\alpha)^3}, \bar{\lambda}_\alpha \frac{1-(\bar{\lambda}_\alpha \bar{t}_\alpha)^4}{1-(\bar{\lambda}_\alpha \bar{t}_\alpha)^3}]$$

The results are shown in Table 1. In this table, average converted service time t from Formulation (11) and the average arrival rate of the system $\tilde{\lambda}$ are considered as inputs and the obtained publicity level $P_{Mw_\alpha}$and S_{Mw_α}maximized profit as outputs. By considering the data in Table 1 we can apply models (8) and (9) for evaluating the efficiency and target values (formulation(10)) for each $\alpha -$cut as DMU. The results are shown in Table 2.

Table 1: Data for five DMUs with two inputs and two outputs

DMU	Input	Input	Output	Output
α	t	λ_α	P_{Mw_α}	S_{Mw_α}
0	[3,5]	[0.265,0.442]	[0.320,1.031]	[0.414,0.536]
0.25	[3.25,4.75]	[0.279,0.408]	[0.355,0.851]	[0.473,0.536]
0.5	[3.5,4.5]	[0.295,0.379]	[0.399,0.713]	[0.510,0.536]
0.75	[3.75,4.25]	[0.312,0.353]	[0.453,0.605]	[0.530,0.536]
1	4	0.331	0.520	0.536

Table 2: The interval efficiencies and targets for the five DMUs

DMU	Efficiency	Target Input	Target Input	Target Input	Target Input
α	$[\theta^{*L}, \theta^{*U}]$	t	λ_α	P_{Mw_α}	S_{Mw_α}
0	[0.600,1.000]	3	0.265	1.031	0.536
0.25	[0.650,0.950]	3	0.265	1.031	0.536
0.5	[0.699,0.898]	3	0.265	1.031	0.536
0.75	[0.751,0.849]	3	0.265	1.031	0.536
1	0.801	3	0.265	1.031	0.536

According to the results provided in Table 2 the DM or manager can find the DMU $\alpha = 0$ is more efficient and can be chosen as the ideal for the other DMUs. Therefore, if the system has average time, $t = 3$; average arrival rate to the system, $\lambda = 0.256$; level of expense in publicity $\mathrm{P}_{\mathrm{Mw}} = 1.031$ and standardized profit $\mathrm{S}_{\mathrm{Mw}_{\alpha}} = 0.536$, it can work more efficiently than the other situations. This example can be considered as the first step for the relation between queueing model and data envelopment analysis. Hope as the future research much more comprehensive application of DEA into queueing models with publicity and renouncement can be developed.

5. Conclusions

In this paper a new approach was provided for the fuzzy queueing models with publicity and renouncement. In the existing models, in the last stage, we can compute the average arrival rate of the system, level of expense in publicity and function of standardized profit in the word of mouth model among the different α −level set. In such a case, The DM or a manager who supervises this system may desire to know which α −level set is working more efficiently than the others. In this paper, a second stage was presented to evaluate the relative efficiency of this word of mouth model. To deal with this situation, an interval DEA model is proposed such that each α −level set is supposed as a decision making unit to be evaluated. The average rate into the system and average service time are inputs and the level of expense in publicity and function of standardized profit are outputs. With the numerical results, the suggested approach is illustrated for the evaluation of the overall efficiency of fuzzy queueing models with publicity and renouncement. This attitude can be considered as the first step for the application of DEA models in the fuzzy queueing system. Hopefully, in the future, more comprehensive researches will be developed toward this subject.

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