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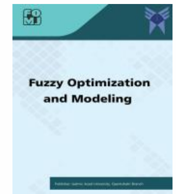

Rough Set in Fuzzy Approximation Space with Fuzzy Equivalence Class

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ABSTRACT

The rough set model was constructed in fuzzy approximation space. In this study, we first introduce the fuzzy relation, relative sets, and fuzzy equivalence class. Then, we prove some properties of the fuzzy equivalence class. Thereafter, the concept of fuzzy rough set is proposed over fuzzy relation and inverse fuzzy relation in fuzzy approximation space by means of relative sets and fuzzy equivalence class sets, and some propositions are proved. Also, some examples and dentitions are presented in this study.

1. Introduction

The notion of fuzzy set was introduced by the celebrated scientist L.A. Zadeh [15] in the year 1965. One of the most important concepts for rough sets is relation. Rosenfeld in [9] defined the fuzzy relation while Adabitabar Frozja and Firouzian [1] proposed the fuzzy number valued fuzzy relation. Fuzzy approximation space is defined with fuzzy relation (see [8]). The rough set was introduced by Pawlak [7] where constructed in fuzzy approximation space and due to its wide use, especially in decision making; it has gained the attention of numerous researchers [10, 4, 14, 12, 3]. Yan et al. [14] introduced the concept of rough set over dual-universe sets in fuzzy approximation space. In a study of rough set theory by Wu et al. [12], the axiomatic characterizations of approximation operators were proposed. Tiwari et al. [11] investigate relationship among fuzzy rough sets, fuzzy closure spaces and fuzzy topology.

Some researches such as Wang [13] have worked on type-2 fuzzy rough sets. In addition, the intuitionistic fuzzy rough set model has been proposed by some researchers such as Liu et al. [5]. Acharjya [2] presented comparative study of rough sets on fuzzy approximation spaces and intuitionistic fuzzy approximation spaces. In this study, a background of fuzzy concepts is presented in Section 2. In Section 3, we introduce the fuzzy relation, relative sets and also fuzzy equivalence relation and fuzzy equivalence class on fuzzy relation with examples. The fuzzy rough set on fuzzy relation with some propositions and examples are presented in Section 4. Finally, the conclusions are presented in Section 5.

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2. Background

A fuzzy subset $\tilde{A}$ of X is a mapping $\mu_A : X \to [0,\ 1]$ where μ_A as assigning to each element $x \in X$ a degree of membership, $0 \le \mu_A(x) \le 1$.

Let X be a set and $\tilde{A}$ and $\tilde{B}$ be fuzzy subsets of X:

(1) A fuzzy subset $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_A(x) \le \mu_B(x)$ for all $x \in X$.
(2) A fuzzy subset ϕ is defined as $\phi = \{(x, \mu_\phi(x)) \mid \mu_\phi(x) = 0\}$ for all $x \in X$.
(3) $\tilde{A} \cup \tilde{B} = \left\{\left(x, \mu_{A\cup B}(x)\right) \middle| \mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x)\right\}$ for all $x \in X$.
(4) $\tilde{A} \cap \tilde{B} = \left\{\left(x, \mu_{A\cap B}(x)\right) \middle| \mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x)\right\}$ for all $x \in X$.
(5) $\widetilde{A} \times \widetilde{B} = \{((x,y), \mu_{A\times B}(x,y)) | \mu_{A\times B}(x,y) = \mu_A(x) \wedge \mu_B(y)\}$.
Here, max and min are show with $\vee$ and $\wedge$ respectively.
(6) $[\tilde{A}]_\alpha = \{x \in X \mid \mu_A(x) \ge \alpha\}$ for all $\alpha \in [0,\ 1]$.
(7) $Supp(\tilde{A}) = \{x \in X \mid \mu_A(x) > 0\}$.

3. Fuzzy Relation and Relative sets

Rosenfeld [9] defined the fuzzy relation, but we present fuzzy relation with some property as follows.

Definition 1. Let $\tilde{A}$ and $\tilde{B}$ be two finite and nonempty fuzzy sets. A binary fuzzy relation $F(\tilde{A}, \tilde{B})$ is fuzzy set in $\tilde{A} \times \tilde{B}$. $F(\tilde{A}, \tilde{B})$ is defined as

$$F(\tilde{A}, \tilde{B}) = \left\{\left((x,y), \mu_F\left((x,y)\right)\right) \middle| \mu_F\left((x,y)\right) \le \mu_A(x) \wedge \mu_B(y)\right\} \tag{1}$$

Define α-cut relation of F as

$$F_\alpha(\tilde{A}, \tilde{B}) = \{(x,y) | \mu_F((x,y)) \ge \alpha\}$$

If $\tilde{A}_1 \subseteq \tilde{A}$ then fuzzy relative set is defined as,

$$F(\tilde{A}_1) = \left\{\left(y, \mu_{F(A_1)}(y)\right) \middle| \mu_{F(A_1)}(y) = \bigvee_x \{\mu_{A_1}(x) \wedge \mu_F(x,y)\}\right.$$

Define α-cut fuzzy relative set of $F(\tilde{A}_1)$ as

$$F_\alpha(\tilde{A}_1) = \{y \mid \mu_{(A_1)}(x) \wedge \mu_F(x,y) \ge \alpha\}$$

If $\tilde{A}_1 = \{(x, \mu_A(x))\}$ then we show fuzzy relative set by $F(x)$ where

$$F(\tilde{A}_1) = \left\{\left(y, \mu_{F(x)}(y)\right) \middle| \mu_{F(x)}(y) = \mu_F(x,y)\right\} = F(x)\ ,\ F_\alpha(\tilde{A}_1) = \{y | \mu_F(x,y) \ge \alpha\} = F_\alpha(x).$$

Also, $\acute{F}$, the inverse relation of F is defined as

$$\acute{F}\left(\tilde{A}, \tilde{B}\right) = \{((y,x), \mu_{\acute{F}}(y,x)) | \mu_{\acute{F}}(y,x) = \mu_F(x,y)\}, \tag{2}$$

If $\tilde{B}_1 \subseteq \tilde{B}$, then fuzzy inverse relative set,

$$F(\tilde{B}_1) = \{(x, \mu_{\acute{F}(B_1)}(x)) \mu_{\acute{F}(B_1)}(x) = \vee_y \{\mu_{B_1}(y) \wedge \mu_F(x,y)\}.$$

If $\tilde{B}_1 = \{(y, \mu_B(y))\}$ then we show fuzzy inverse relative set by $\acute{F}(y)$ where:

$$\acute{F}(y) = \left\{\left(x, \mu_{\acute{F}(y)}(x)\right) \middle| \mu_{\acute{F}(y)}(x) = \mu_F(x,y)\right\}.$$

Example 1. Let $\tilde{A} = \{(a_1, 0.1), (a_2, 1), (a_3, 0.4), (a_4, 0.9)\}$ and $\tilde{B} = \{(b_1, 0.6), (b_2, 0.7), (b_3, 1)\}$ and $\tilde{A}_1 = \{(a_2, 0.6), (a_3, 0.4)\}$ and $\tilde{B}_1 = \{(b_2, 0.7)\}$ then

$$\tilde{A} \times \tilde{B} = \{((a_1,b_1),0.1), ((a_1,b_2),0.1), ((a_1,b_3),0.1), ((a_2,b_1),0.6), ((a_2,b_2),0.7), ((a_2,b_3),1), ((a_3,b_1),0.4), ((a_3,b_2),0.4), ((a_3,b_3),0.4), ((a_4,b_1),0.6), ((a_4,b_2),0.7), ((a_4,b_3),0.9)\}.$$

If

$F(\tilde{A}, \tilde{B}) = \{((a_1,b_1),0.1), ((a_2,b_1),0.4), ((a_2,b_3),0.8), ((a_3,b_2),0.4), ((a_3,b_3),0.2), ((a_4,b_1),0.5), ((a_4,b_2),0.4),$

$((a_4,b_3),0.9)\}$, then
$F_{0.6}(\tilde{A},\tilde{B})=\{(a_2,b_3),(a_4,b_3)\}$, $F_{0.5}(\tilde{A},\tilde{B})=\{(a_2,b_3),(a_4,b_1),(a_4,b_3)\}$, $F(\tilde{A}_1)=\{(b_1,0.4),(b_2,0.4),(b_3,0.6)\}$, $F_{0.5}(\tilde{A}_1)=\{b_3\}$, $\acute{F}(\tilde{B}_1)=\{(a_3,0.4),(a_4,0.4)\}$.
Mordeson [6] proposed the reflexive, symmetric and transitive of fuzzy relation. Following definition is a generalization of above definition. We discuss the difference of two definitions.

Definition 2. If $\tilde{A}$ is fuzzy set then fuzzy relation $F(\tilde{A},\tilde{A})$ define as

$$F(\tilde{A},\tilde{A})=\{((x,y),\mu_F((x,y)))\mid \mu_F((x,y))\le \mu_A(x)\wedge\mu_A(y)\} \tag{3}$$

(1) If $\forall x;\ (0<\mu_F((x,x))<\mu_A(x))\ \mu_F((x,x))=\mu_A(x)$ then F is (week reflexive) reflexive.
(2) If $\forall x,y;\ (\mu_F((x,y))>0,\mu_F((y,x))>0)\ \mu_F((x,y))=\mu_F((y,x))=\mu_A(x)\wedge\mu_A(y)$ then F is (week symmetric) symmetric.
(3) If $\forall x,y,z;\ \left(\mu_F((x,y))>0,\mu_F((y,z))>0\Rightarrow 0<\mu_F((x,z))\le\mu_F((x,y))\wedge\mu_F((y,z))\right),\mu_F((x,z))=\mu_F((x,y))\wedge\mu_F((y,z))=\mu_A(x)\wedge\mu_A(y)\wedge\mu_A(z)$ then F is (week transitive) transitive.

Definition 3. Fuzzy relation F on $\tilde{A}$ which (week reflexive, week symmetric and week transitive) reflexive, symmetric and transitive is called a (week fuzzy equivalence relation) fuzzy equivalence relation on $\tilde{A}$. We define fuzzy equivalence (week fuzzy equivalence) class respect to F as follows:

$$[\tilde{x}]_F=\{(y,\mu_{[x]F}(y))|\mu_{[x]F}(y)=\mu_F((y,y)), where\ \mu_F((y,y))\le\mu_F((y,x))\wedge\mu_F((x,y))\}.$$

Proposition 1. *If F is a fuzzy equivalence relation as crisp case $[\tilde{x}]_F=F(x)$.*

Proof. It is clear from Definitions 1 and 3.

Proposition 2. *If $[\tilde{x}]_F\cap[\tilde{y}]_F\ne\emptyset$ then $[\tilde{x}]_F=[\tilde{y}]_F$.*

Proof. We consider two cases.
Case 1. F is a fuzzy equivalence relation then $F(x)=[\tilde{x}]_F=\{(t,\mu_{[x]F}(t))|\mu_{[x]F}(t)=\mu_F((t,x))\}$ If $[\tilde{x}]_F\cap[\tilde{y}]_F\ne\emptyset$ then $\exists t:\ \mu_{[x]F}(t)>0,\ \mu_{[y]F}(t)>0$. But, $\mu_{[x]F}(t)=\mu_F((t,x))=\mu_F((x,t))=\mu_F((t,t))\}$ and $\mu_{[y]F}(t)=\mu_F((t,y))=\mu_F((y,t))=\mu_F((t,t))\}$. Hence $\mu_{[x]F}(t)=\mu_{[y]F}(t)$ therefor $[\tilde{x}]_F=[\tilde{y}]_F$.

Case 2. F is a week fuzzy equivalence relation then $[\tilde{x}]_F=\{(t,\mu_{[x]F}(t))|\mu_{[x]F}(t)=\mu_F((t,t),$ where $\mu_F((t,t))\le\mu_F((t,x))\wedge\mu_F((x,t))\}$.
If $[\tilde{x}]_F\cap[\tilde{y}]_F\ne\emptyset$ then $\exists t:\ \mu_{[x]F}(t)>0,\ \mu_{[y]F}(t)>0$. But, $\mu_{[x]F}(t)=\mu_F((t,t)),\ \mu_F((t,t))\le\mu_F((t,x))\wedge\mu_F((x,t))\}$ and $\mu_{[y]F}(t)=\mu_F((t,t)),\ \mu_F((t,t))\le\mu_F((t,y))\wedge\mu_F((y,t))\}$, and hence $\mu_{[x]F}(t)=\mu_{[y]F}(t)$. Therefor $[\tilde{x}]_F=[\tilde{y}]_F$.

Proposition 3. $supp(\cup_x[\tilde{x}]_F)=supp(\tilde{A})$**.**

Proof. $t\in supp(\cup_x[\tilde{x}]_F)\mapsto\exists x:\mu_{[x]}(t)>0$. If F is a week fuzzy equivalence relation then $\mu_{[x]}(t)=\mu_F(t,t)\le\mu_A(t)$ hence $\mu_A(t)>0$ therefor $t\in supp(\tilde{A})$ and $supp(\cup_x[\tilde{x}]_F)\subseteq supp(\tilde{A})$. If F is a fuzzy equivalence relation then $\mu_{[x]}(t)=\mu_F(t,x)=\mu_A(t)\wedge\mu_A(x)$ hence$\mu_A(t)>0$ therefor $t\in supp(\tilde{A})$ and $supp(\cup_x[\tilde{x}]_F)\subseteq supp(\tilde{A})$. Vice versa, $t\in supp(\tilde{A})$ then $\exists x:\ \mu_{[x]}(t)>0\mapsto t\in supp(\cup_x[\tilde{x}]_F)$ hence $supp(\tilde{A})\subseteq supp(\cup_x[\tilde{x}]_F)$. So proof is compleat.

Example 2. Let $\tilde{A}=\{(a_1,0.1),(a_2,1),(a_3,0.4),(a_4,0.9)\}$ and $F(\tilde{A},\tilde{A})=\{((a_1,a_1),0.1),((a_1,a_3),0.1),((a_2,a_2),1),((a_2,a_4),0.9),((a_3,a_1),0.1),((a_3,a_3),0.4),((a_4,a_2),0.9),((a_4,a_4),0.9)\}$. F is reflexive, symmetric and transitive, therefore called a fuzzy equivalence relation on $\tilde{A}$. $[\tilde{a}_1]_F=\{(a_1,0.1),(a_3,0.1)\}$ and $[\tilde{a}_2]_F=\{(a_2,1),(a_4,0.9)\}$.

Example 3. Let $\tilde{A}=\{(a_1,0.1),(a_2,1),(a_3,0.4),(a_4,0.9)\}$ and $(\tilde{A},\tilde{A})=\{((a_1,a_1),0.1),((a_1,a_3),0.1),((a_2,a_2),0.6),((a_2,a4),0.4),((a_3,a_1),0.1),((a_3,a_3),0.2),((a_4,a_2),0.5),((a_4,a_4),0.3)\}$. F is week reflexive, week symmetric and week transitive, therefore called a week fuzzy equivalence relation on $\tilde{A}$. $[\tilde{a}_1]_F=\{(a_1,0.1),(a_3,0.1)\}$ and $[\tilde{a}_2]_F=\{(a_2,0.6),(a_4,0.3)\}$.

4. Fuzzy Rough set on Fuzzy Relation

Yan et al. [14] introduced rough set over dual-universe sets in fuzzy approximation space but, we introduce a new definition of rough set by using membership function.

Definition 4. Let $\widetilde{U}$ and $\widetilde{V}$ be the fuzzy universe of discourse. Assume $F(\widetilde{U},\widetilde{V})$ is fuzzy relation and $\acute{F}(\widetilde{V},\widetilde{U})$ is the inverse fuzzy relation. For $\forall\, \widetilde{Y} \subseteq \widetilde{V}$, the lower and upper approximation of $\widetilde{Y}$ with respect to F are define as

$$\begin{aligned} \underline{\acute{F}}\widetilde{Y} &= \{(x, \mu_{\underline{\acute{F}Y}}(x)) \mid \mu_{\underline{\acute{F}Y}}(x) = \mu_F((x,y)), \mu_Y(y) > 0, \mu_F((x,y)) \le \mu_Y(y)\} \\ \overline{\acute{F}}\widetilde{Y} &= \left\{\left(x, \mu_{\overline{\acute{F}Y}}(x)\right) \middle| \mu_{\overline{\acute{F}Y}}(x) = \mu_F\big((x,y)\big), \mu_F\big((x,y)\big) > 0, \mu_Y(y) > 0\right\}. \end{aligned} \tag{4}$$

Or

$$\begin{aligned} \underline{\acute{F}}\widetilde{Y} &= \cup \{\acute{F}(y) \mid \mu_F((x,y)) \le \mu_Y(y), \mu_Y(y) > 0\} \\ \overline{\acute{F}}\widetilde{Y} &= \cup \{\acute{F}(y) \mid \mu_F((x,y)) > 0, \mu_Y(y) > 0\} \end{aligned} \tag{5}$$

where $\acute{F}(y)$ is fuzzy inverse relative set.

For $\forall\ \widetilde{X} \subseteq \widetilde{U}$, the lower and upper approximation of $\widetilde{X}$ with respect to F are define as

$$\begin{aligned} \underline{F}\,\widetilde{X} &= \{(y, \mu_{\underline{FX}}(y)) \mid \mu_{\underline{FX}}(y) = \mu_F((x,y)), \mu_X(x) > 0, \mu_F((x,y)) \le \mu_X(x)\} \\ \overline{F}\,\widetilde{X} &= \{(y, \mu_{\overline{FX}}(y)) \mid \mu_{\overline{FX}}(y) = \mu_F((x,y)), \mu_F((x,y)) > 0, \mu_X(x) > 0\} \end{aligned} \tag{6}$$

Or

$$\begin{aligned} \underline{F}\,\widetilde{X} &= \cup \{F(x) \mid \mu_F((x,y)) \le \mu_X(x), \mu_X(x) > 0\} \\ \overline{F}\,\widetilde{X} &= \cup \{F(x) \mid \mu_F((x,y)) > 0, \mu_X(x) > 0\} \end{aligned} \tag{7}$$

where $F(x)$ is fuzzy relative set.

Proposition 4. Let $\widetilde{U}$ and $\widetilde{V}$ be the fuzzy universe of discourse. Assume $F(\widetilde{U},\widetilde{V})$ is fuzzy relation and $\acute{F}(\widetilde{V},\widetilde{U})$ is inverse fuzzy relation. For $\forall\, \widetilde{Y} \subseteq \widetilde{V}$ and $\forall\, \widetilde{X} \subseteq \widetilde{U}$,

(1) If $\widetilde{Y} \subseteq \widetilde{V}$ then $\underline{\acute{F}}\,\widetilde{Y} \subseteq \overline{\acute{F}}\widetilde{Y}$,

(2) If $\widetilde{X} \subseteq \widetilde{U}$ then $\underline{F}\,\widetilde{X} \subseteq \overline{F}\widetilde{X}$,

(3) If $\widetilde{Y}_1 \subseteq \widetilde{Y}_2$ then $\underline{\acute{F}}\,\widetilde{Y}_1 \subseteq \underline{\acute{F}}\,\widetilde{Y}_2$ and $\overline{\acute{F}}\,\widetilde{Y}_1 \subseteq \overline{\acute{F}}\widetilde{Y}_1$,

(4) If $\widetilde{X}_1 \subseteq \widetilde{X}_2$ then $\underline{F}\widetilde{X}_1 \subseteq \underline{F}\widetilde{X}_2$ and $\overline{F}\widetilde{X}_1 \subseteq \overline{F}\widetilde{X}_2$.

Proof. Regarding to (5) and (7) properties of (1) and (2) are evident. For property of (3), $\widetilde{Y}_1 \subseteq \widetilde{Y}_2$ then $\mu_{Y_1}(y) \le \mu_{Y_2}(y)$ therefor

$$\begin{aligned} \underline{\acute{F}}\,\widetilde{Y}_1 &= \cup \left\{ \acute{F}(y) \middle| \mu_F\big((x,y)\big) \le \mu_{Y_1}(y), \mu_{Y_1}(y) > 0 \right\} \\ &\subseteq \cup \{\acute{F}(y) \mid \mu_F((x,y)) \le \mu_{Y_2}(y), \mu_{Y_2}(y) > 0\} = \underline{\acute{F}}\,\widetilde{Y}_2. \end{aligned}$$

$$\begin{aligned} \overline{\acute{F}}\,\widetilde{Y}_1 &= \cup \left\{\acute{F}(y) \middle| \mu_F\big((x,y)\big) > 0, \mu_{Y_1}(y) > 0\right\} \\ &\subseteq \cup \{\acute{F}(y) \mid \mu_F((x,y)) > 0, \mu_{Y_2}(y) > 0\} = \overline{\acute{F}}\,\widetilde{Y}_2. \end{aligned}$$

For property of (4), $\widetilde{X}_1 \subseteq \widetilde{X}_2$ then $\mu_{X_1}(x) \le \mu_{X_2}(x)$ therefor

$$\begin{aligned} \underline{F}\widetilde{X}_1 &= \cup \left\{F(x) \middle| \mu_F\big((x,y)\big) \le \mu_{X_1}(x), \mu_{X_1}(y) > 0\right\} \\ &\subseteq \cup \{F(x) \mid \mu_F((x,y)) \le \mu_{X_2}(x), \mu_{X_2}(x) > 0\} = \underline{F}\widetilde{X}_2. \end{aligned}$$

$$\begin{aligned} \overline{F}\widetilde{X}_1 &= \cup \left\{F(x) \middle| \mu_F\big((x,y)\big) > 0, \mu_{X_1}(x) > 0\right\} \\ &\subseteq \cup \{F(x) \mid \mu_F((x,y)) > 0, \mu_{X_2}(x) > 0\} = \overline{F}\widetilde{X}_2. \end{aligned}$$

Definition 4. Let $\widetilde{U}$ be the fuzzy universe of discourse. Assume F is fuzzy relation on $\widetilde{U}$. For $\forall\, \tilde{X} \subseteq \widetilde{U}$, the lower and upper approximation of $\tilde{X}$ with respect to F are define as

$$\underline{F}\,\tilde{X} = \{(y, \mu_{\underline{FX}}(y)) | \mu_{\underline{FX}}(y) = \mu_F((x,y)), \mu_X(x) > 0, \mu_F((x,y)) \leq \mu_X(x)\}$$
$$\overline{F}\,\tilde{X} = \{(y, \mu_{\overline{FX}}(y)) | \mu_{\overline{FX}}(y) = \mu_F((x,y)), \mu_F((x,y)) > 0, \mu_X(x) > 0\} \quad (8)$$

Or

$$\underline{F}\,\tilde{X} = \cup\left\{F(x) \middle| \mu_F\big((x,y)\big) \leq \mu_X(x), \mu_X(x) > 0\right\} = \cup\left\{ [\tilde{x}]_F \middle| [\tilde{x}]_F \subseteq \tilde{X}\right\}$$
$$\overline{F}\,\tilde{X} = \cup\left\{F(x) \middle| \mu_F\big((x,y)\big) > 0, \mu_X(x) > 0\right\} = \cup\left\{ [\tilde{x}]_F \middle| [\tilde{x}]_F \cap \tilde{X} \neq \emptyset\right\} \quad (9)$$

Example 4. Let $\widetilde{U} = \{(a_1, 0.1), (a_2, 1), (a_3, 0.4), (a_4, 0.9), (a_5, 0.3), (a_6, 1), (a_7, 0.8), (a_8, 0.5)\}$ and F be the fuzzy relation on $\widetilde{U}$.

$$\begin{aligned} F(\widetilde{U}, \widetilde{U}) = \{&((a_1,a_1),0.1),((a_1,a_7),0.1),((a_2,a_2),0.8),((a_2,a_5),0.3),((a_2,a_8),0.4),((a_3,a_3),0.3) \\ &\big((a_4,a_4),0.8\big),\big((a_4,a_6),0.8\big),\big((a_5,a_2),0.2\big),\big((a_5,a_5),0.2\big),\big((a_5,a_8),0.2\big),\big((a_6,a_4),0.8\big), \\ &((a_6,a_6),1),((a_7,a_1),0.1),((a_7,a_7),0.7),((a_8,a_2),0.4),((a_8,a_5),0.3),((a_8,a_8),0.4)\} \end{aligned}$$

F is week reflexive, week symmetric and week transitive, therefore called a week fuzzy equivalence relation on $\tilde{A}$.
$[\tilde{a}_1]_F = \{(a_1, 0.1), (a_7, 0.1)\}$,$[\tilde{a}_2]_F = \{(a_2, 0.8), (a_5, 0.2), (a_8, 0.4)\}$. $[\tilde{a}_3]_F = \{(a_3, 0.3)\}$, $[\tilde{a}_4]_F = \{(a_4, 0.8), (a_6, 0.8)\}$.
If $\widetilde{A} = \{(a_1, 0.1), (a_2, 0.6), (a_3, 0.3), (a_7, 0.1)\}$, then

$$\underline{F}\,\widetilde{A} = [\tilde{a}_1]_F \cup [\tilde{a}_3]_F = \{(a_1, 0.1), (a_3, 0.3), (a_7, 0.1)\}$$
$$\overline{F}\,\widetilde{A} = [\tilde{a}_1]_F \cup [\tilde{a}_2]_F \cup [\tilde{a}_3]_F = \{(a_1, 0.1), (a_2, 0.8), (a_3, 0.3), (a_5, 0.2)(a_7, 0.1), (a_8, 0.4)\}.$$

5. Conclusions

In this study, another type of model was developed for the evaluation of rough sets in fuzzy approximation space with fuzzy relation, using relative sets and membership function. Also, in this study, various properties of fuzzy relation, such as reflexive (week reflexive), symmetric (week symmetric) and transitive (week transitive), were proposed and thereafter, the fuzzy equivalence relation (week fuzzy equivalence relation) with fuzzy equivalence class was presented.

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